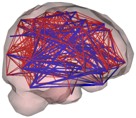
Fractal brain connectivity
Functional connectivity using wavelets and graph theory
Part I: Graphs at a glance

Sophie Achard

CNRS, GIPSA-lab, Grenoble
sophie.achard@gipsa-lab.inpg.fr

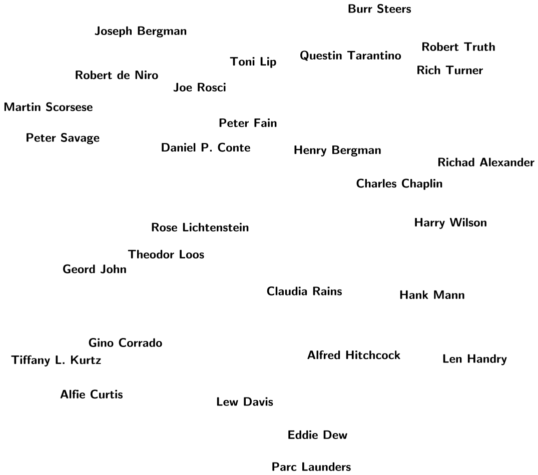
Grenoble, 27 September 2013



Why representing data as complex networks?

Nowadays data:

- High number
- Spatial location
- Common characteristics



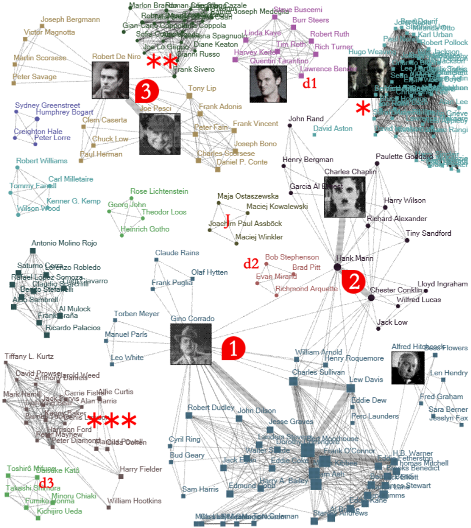
Why representing data as complex networks?

Nowadays data:

- High number
- Spatial location
- Common characteristics

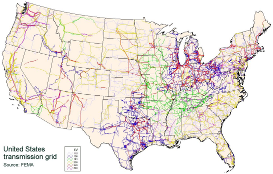
Representation:

- Networks
- Communities

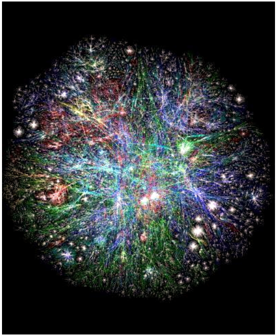


Examples of complex networks in real life

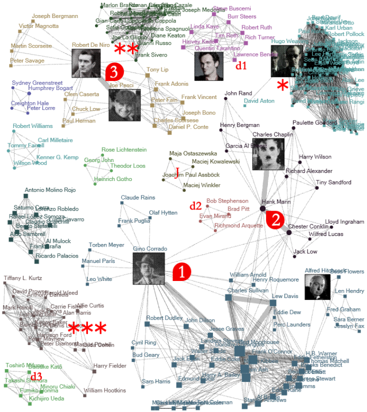
Electric power grid



WWW



Social network

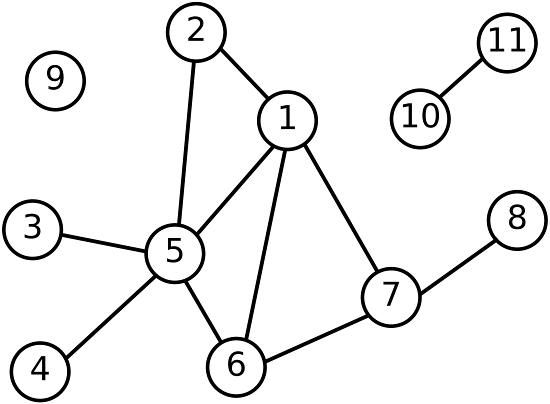


Complex networks or graphs: mathematical definition

Definition

A graph is an abstract representation of a set of objects where some pairs of the objects are connected by links.
A graph is an ordered pair $G = (V, E)$ comprising a set V of vertices or nodes together with a set E of edges or lines, which are 2-element subsets of V (i.e, an edge is related with two vertices, and the relation is represented as unordered pair of the vertices with respect to the particular edge). To avoid ambiguity, this type of graph may be described precisely as undirected and simple.

Complex networks or graphs: mathematical definition

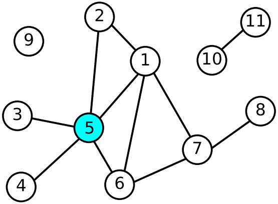


Set of vertices: $V = \{1,2,3,4,5,6,7,8,9,10,11\}$

Set of edges: $E = \{\{1,2\}, \{1,5\},\{1,6\}, \{1,7\}, \{2,5\}, \{3,5\}, \{4,5\}, \{5,6\}, \{6,7\}, \{7,8\}, \{10,11\}\}$

Complex networks or graphs: mathematical definition

A graph can be uniquely defined by the adjacency matrix :



0	1	0	0	1	1	1	0	0	0	0
1	0	0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0
1	1	1	1	0	1	0	0	0	0	0
1	0	0	0	1	0	1	0	0	0	0
1	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1
0	0	0	0	0	0	0	0	0	1	0

Complex networks or graphs: mathematical definition

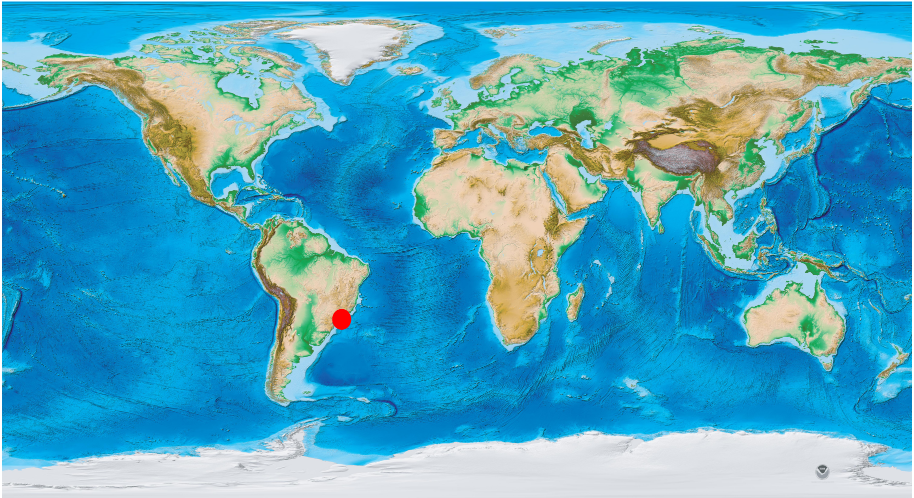
Some classical types of graphs :

- simple or multi
- directed or undirected
- weighted or unweighted

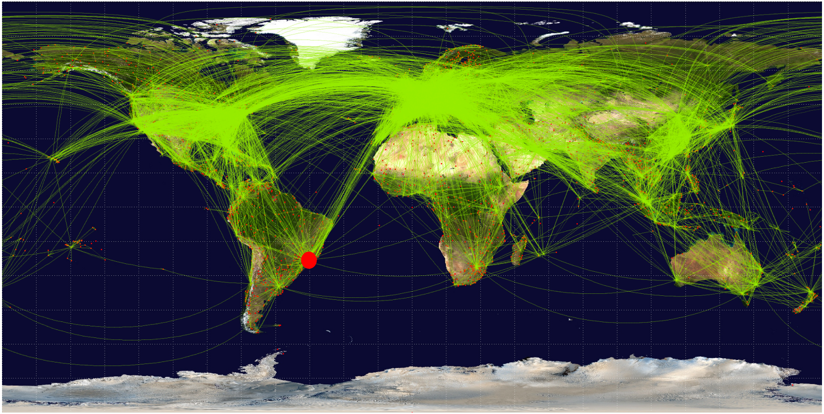
In the sequel, we will always consider simple undirected graphs.

Some references [Bollobás, 1998; Whittaker, 1990; Diestel, 2005].

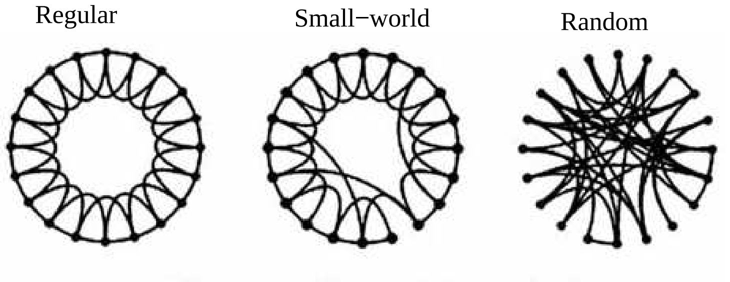
Why using graphs: the epidemiology example



Why using graphs: the epidemiology example



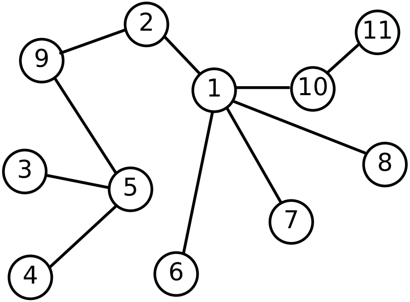
Comparisons of graphs: the small-world idea

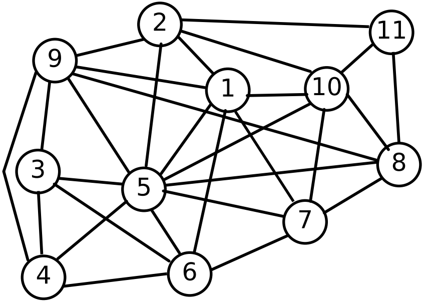


[Watts and Strogatz 1998]

In practice: graph metrics

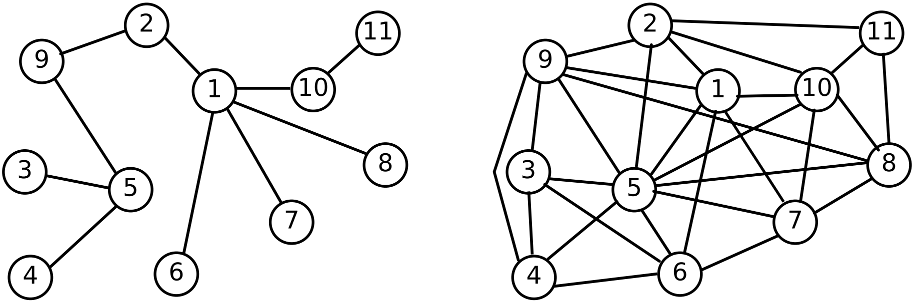
A graph is still a multivariate representation of the data. One should summarize them in some sense.





In practice: graph metrics

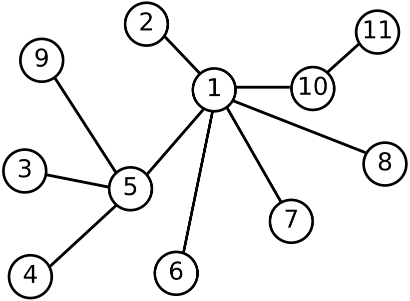
A graph is still a multivariate representation of the data. One should summarize them in some sense.

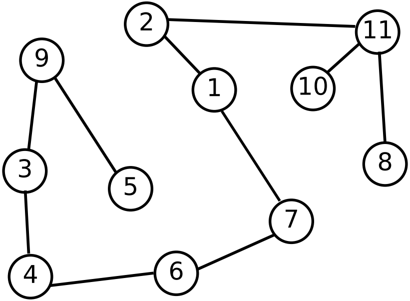


Degree: the number of connections that node makes to other nodes in the graph

In practice: graph metrics

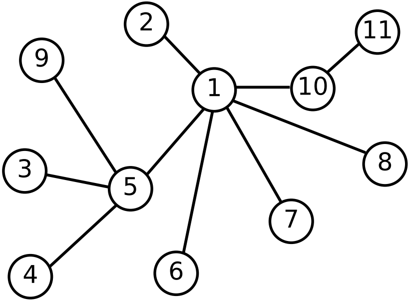
A graph is still a multivariate representation of the data. One should summarize them in some sense.

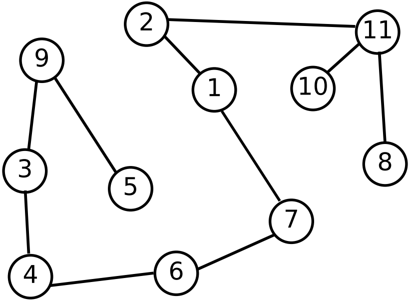




In practice: graph metrics

A graph is still a multivariate representation of the data. One should summarize them in some sense.

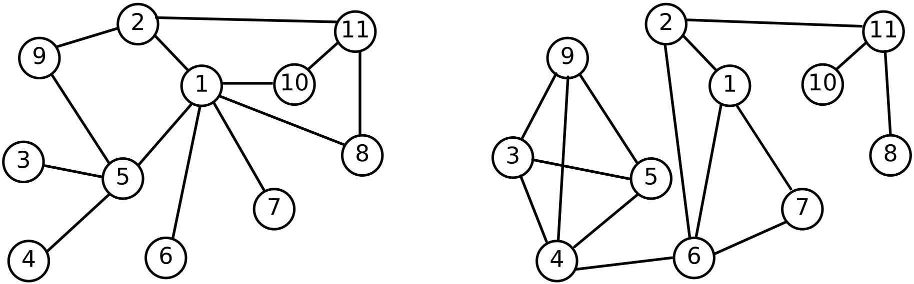




The **global efficiency** measures how the information is propagating in the whole network.

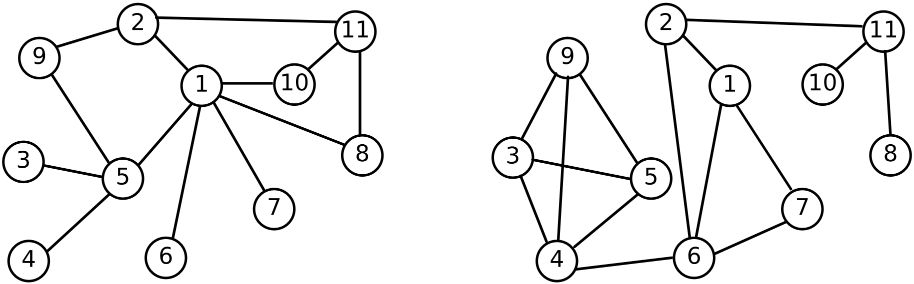
In practice: graph metrics

A graph is still a multivariate representation of the data. One should summarize them in some sense.



In practice: graph metrics

A graph is still a multivariate representation of the data. One should summarize them in some sense.



Clustering, also called "local efficiency", can be regarded as a measure of information transfer in the immediate neighbourhood of each node.