Fractal brain connectivity Functional connectivity using wavelets and graph theory Part I: Graphs at a glance

Sophie Achard

CNRS, GIPSA-lab, Grenoble sophie.achard@gipsa-lab.inpg.fr

Grenoble, 27 September 2013







Sophie Achard (CNRS, Grenoble)

Fractal brain connectivity



1 What are complex networks?





# Why representing data as complex networks?

|                                      |                                  | Burr Stee                | Burr Steers          |                             |  |  |
|--------------------------------------|----------------------------------|--------------------------|----------------------|-----------------------------|--|--|
|                                      | Joseph Bergman                   | 1                        | . Questin Tarantino  | Robert Truth                |  |  |
|                                      | Robert de Niro                   | Toni Li<br>pe Rosci      | ip Questin Tarantino | Rich Turner                 |  |  |
|                                      | Martin Scorsese                  |                          |                      |                             |  |  |
|                                      | Peter Savage Dani                | Peter Fa<br>iel P. Conte | ın<br>Henry Bergman  |                             |  |  |
| Nowadays data:                       |                                  |                          | Charles              | Richad Alexander<br>Chaplin |  |  |
| <ul> <li>High number</li> </ul>      | Rose I                           | Lichtenstein             |                      | Harry Wilson                |  |  |
| <ul> <li>Spatial location</li> </ul> | Theodor L<br>Geord John          | .00S                     |                      |                             |  |  |
| • Common characteristics             |                                  |                          | Claudia Rains I      | Hank Mann                   |  |  |
|                                      | Gino Corrado<br>Tiffany L. Kurtz |                          | Alfred Hitchcock     | Len Handry                  |  |  |
|                                      | Alfie Curtis                     | Lew Davi                 | s                    |                             |  |  |
|                                      |                                  |                          | Eddie Dew            |                             |  |  |
|                                      |                                  | Parc Launders            |                      |                             |  |  |

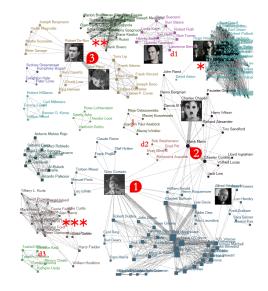
# Why representing data as complex networks?

#### Nowadays data:

- High number
- Spatial location
- Common characteristics

Representation:

- Networks
- Communities



# Examples of complex networks in real life

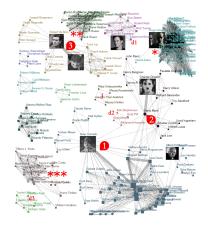
### Electric power grid



WWW



#### Social network



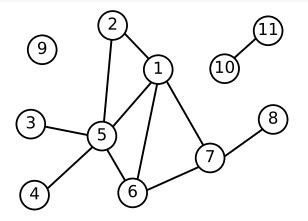
#### Fractal brain connectivity

#### Definition

A graph is an abstract representation of a set of objects where some pairs of the objects are connected by links.

A graph is an ordered pair G = (V, E) comprising a set V of vertices or nodes together with a set E of edges or lines, which are 2-element subsets of V (i.e, an edge is related with two vertices, and the relation is represented as unordered pair of the vertices with respect to the particular edge). To avoid ambiguity, this type of graph may be described precisely as undirected and simple.

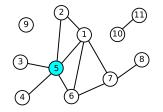
# Complex networks or graphs: mathematical definition



Set of vertices:  $V = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ Set of edges:  $E = \{\{1,2\}, \{1,5\}, \{1,6\}, \{1,7\}, \{2,5\}, \{3,5\}, \{4,5\}, \{5$  $\{5,6\}, \{6,7\}, \{7,8\}, \{10,11\}\}$ 

# Complex networks or graphs: mathematical definition

A graph can be uniquely defined by the adjacency matrix :



| 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |  |
|---|---|---|---|---|---|---|---|---|---|---|--|
| 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |  |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |  |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |  |

# Complex networks or graphs: mathematical definition

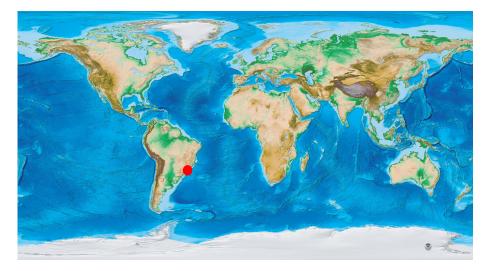
Some classical types of graphs :

- simple or multi
- directed or undirected
- weighted or unweighted

In the sequel, we will always consider simple undirected graphs.

Some references [Bollobás, 1998; Whittaker, 1990; Diestel, 2005].

# Why using graphs: the epidemiology example



Sophie Achard (CNRS, Grenoble)

Fractal brain connectivity

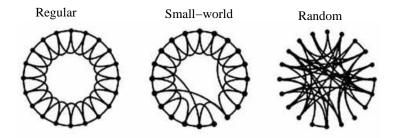
27/09/2013 6 / 10

# Why using graphs: the epidemiology example



Sophie Achard (CNRS, Grenoble)

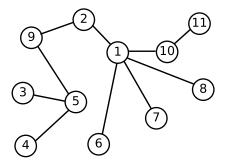
# Comparisons of graphs: the small-world idea

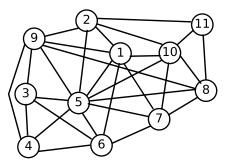


[Watts and Strogatz 1998]

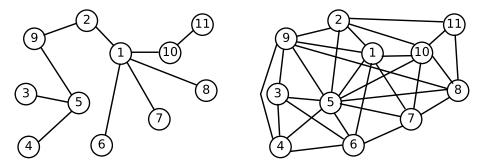
Sophie Achard (CNRS, Grenoble)

A graph is still a multivariate representation of the data. One should summarize them in some sense.



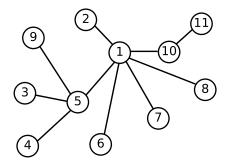


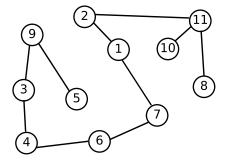
A graph is still a multivariate representation of the data. One should summarize them in some sense.



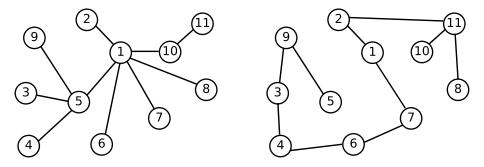
**Degree**: the number of connections that node makes to other nodes in the graph

A graph is still a multivariate representation of the data. One should summarize them in some sense.



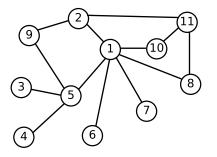


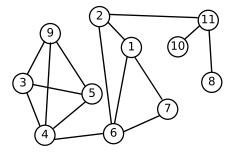
A graph is still a multivariate representation of the data. One should summarize them in some sense.



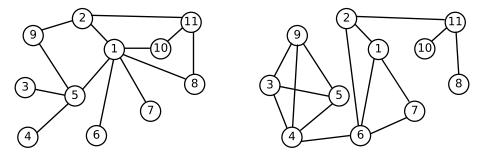
The **global efficiency** measures how the information is propagating in the whole network.

A graph is still a multivariate representation of the data. One should summarize them in some sense.





A graph is still a multivariate representation of the data. One should summarize them in some sense.



**Clustering**, also called "local efficiency", can be regarded as a measure of information transfer in the immediate neighbourhood of each node.